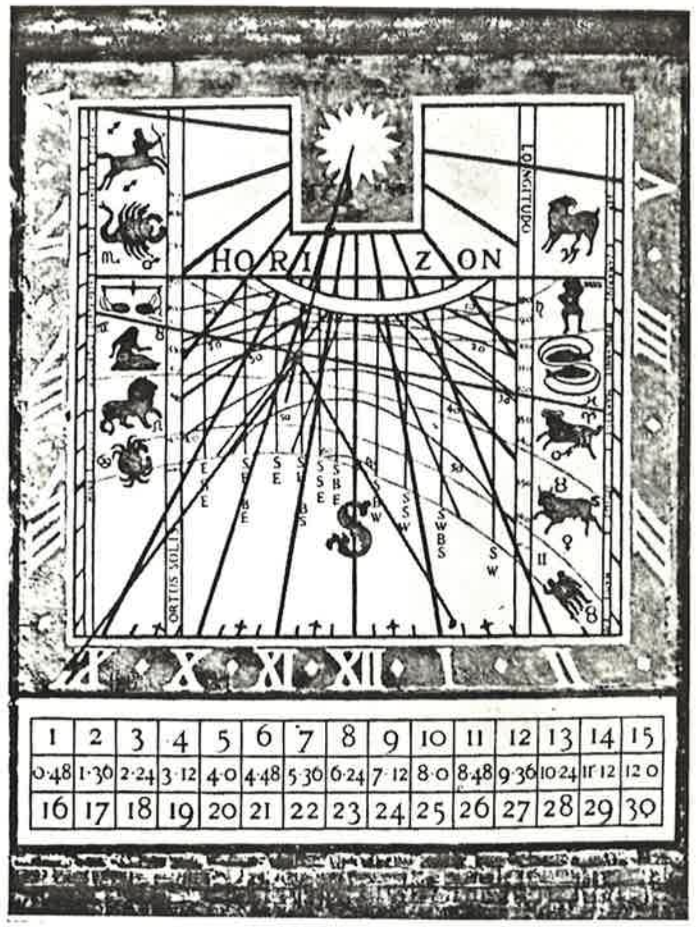


FIG. 34. Vertical declining dial at Queens' College, Cambridge. Often called Newton's dial but painted after Newton's death *c.* 1727. There is no evidence that Sir Isaac had any association with it.

Roman numerals in gold on an outer border of blue give the hours (local apparent time). A set of vertical black lines marked with the compass points give the Azimuth of the Sun. Red curves of hyperbolic form give the Altitude of the Sun. Green curves of hyperbolic form give the Zodiacal position of the Sun, the date, the time of Sunrise and the right ascension of the Sun.

The Dial is also a Moon dial. This construction is supervacaneous since the Moon's change of phase seldom allows it to give a clear shadow. The table below the dial gives figures for the Moon's hour angle for each of 30 days of the month. Local apparent time equals the time shown by the Moon's shadow plus the Moon's hour angle taken from the middle line of the table.

VERTICAL DECLINING[1] DIALS

Vertical declining dials is the term applied to dials fitted to walls which face east and west of true north and south. The actual declination of any wall is readily found by means of a flat square board on to which is fixed a small vertical gnomon.[2] If the shadow of the gnomon at local apparent noon[3] is marked on the board then the angle between this shadow and a normal to the wall is the required declination *D*.

The style extended passes through the celestial poles and its

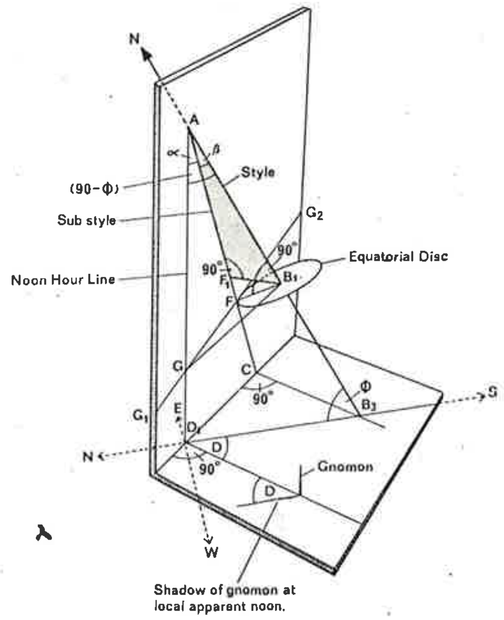


FIG. 35.

[1] Decline – not to be confused with the declination of an astronomical body such as the Sun. Here the term is used simply in the basic sense; viz, to turn aside or deviate from the meridian.

[2] This is tautological since a gnomon is a vertical column. The emphasis is, however, not without importance here. Architects may care to consult British Patent 344337 which shows an 'orientator' or architect's solar dial an instrument for ascertaining from architect's plans the direction of the Sun's rays at different times of the day.

[3] Local apparent noon is obtained from clock time by making two corrections: one for longitude east or west of the time meridian, the other for the equation of time for the date of the observation.

plane is not in the meridian. The arrangement for any station will be similar to the example chosen for illustration in FIG. 35.

GRAPHICAL AND PART GRAPHICAL CONSTRUCTIONS

The noon hour line will be a vertical through the point where the style joins the dial plate. The graphical construction is a little more complex than that of the vertical north/south dials but the graphical layout follows the same plan established in the previous chapter. The equatorial dial or disc is placed at the

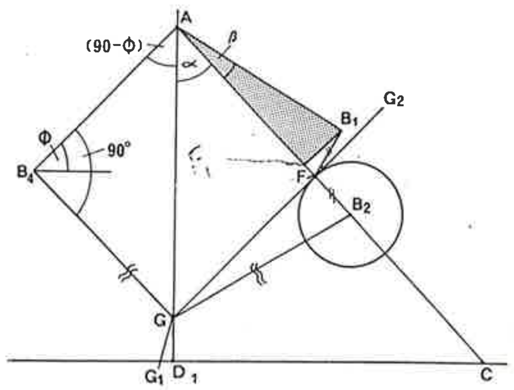


FIG. 35A.

extremity of the theoretical style, FIG. 35 and it is again rabatted downward into the plane of the paper whereas the plane of the style is rabatted to the right or left about the line of the substyle AC extended.

The graphical construction which follows from the three dimensional array is shown in FIG. 35A. The style AB_1 is part of the triangle AB_1F. The true position of the style in space is AB_1B_3 (FIG. 35) making an angle equal to the latitude ϕ with the horizontal plane and an angle $(90 - \phi)$ with the vertical plane of the wall. The noon hour line is shown as AGD_1. The equatorial disc is in its true position with B_1F as radius and $\angle AB_1F$ equal to a right angle. The noon line on the equatorial dial and in its plane extended is shown as B_1G. The substyle is AF_1. The style is now rabatted into the wall along the substyle (hinge) AF_1. The equatorial circle is rabatted downward into the plane of the wall

along the equinoctial line G_1GFG_2 which is normal to AF. Clearly with such a conversion, when the rabattment is complete, FB_1 appears at two positions (FB_1 and FB_2) in the two dimensional space of diagram, Fig. 35A. Again the true style AB_1 may be rabatted to the left about the (hinge) line AG to produce the triangle AB_4G.

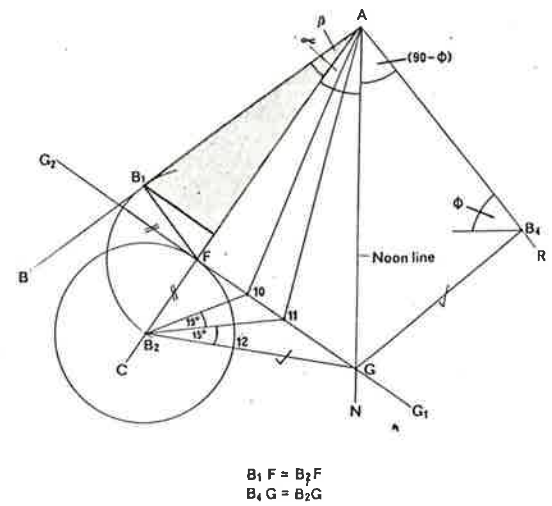


FIG. 36.

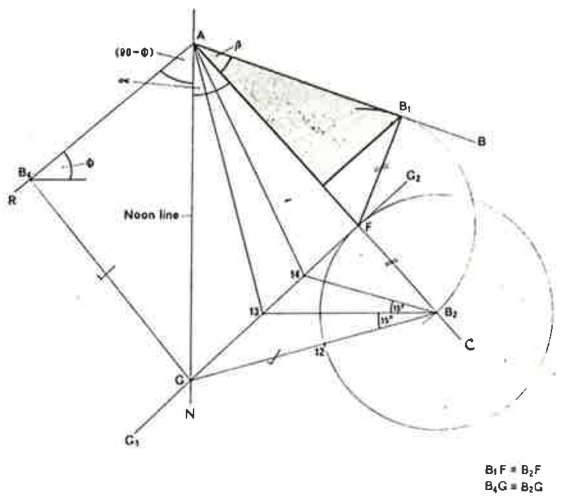


FIG. 37.

Two worked examples (part mathematical and part graphical) will now be given: one for a vertical dial declining East of South in a North latitude, FIG. 36, the other declining West of South, FIG. 37.

We may proceed as follows. We always know the following: The Declination (D) and the Latitude (ϕ).

It is easy to calculate the two angles α and β shown in FIG. 35. α is the angle of the substyle with the noon line and β is the angle of the style with the substyle. The angles α and β may be readily calculated from the equations:

$$\log \tan \alpha = \log \sin D + \log \cot \phi$$
$$\log \sin \beta = \log \cos D + \log \cos \phi$$

We can then lay out the graphical relationships shown in FIGS. 36 and 37. We know the theoretical style length AB_1, we desire, hence we draw the vertical noon line AN and lay out AR at $\angle RAN$ of $(90 - \phi)$; AC at an $\angle NAC = \alpha$ and AB_1 at an $\angle CAB = \beta$. We terminate AB_1 as required to give the style length. From B_1 we draw a normal to AB cutting AC in F. With our compasses set to radius B_1F and centre F we strike an arc cutting AC in B_2. From F normal to AC we draw a line cutting the noon line AN at G. We can now draw G_1G_2 which is the equinoctial line. From G a normal to AR gives point B_4 (and as a check $B_2G = B_4G$). The equatorial circle may now be drawn on point B_2 with radius FB_1 ($FB_1 = FB_2$). The equatorial circle is now sub-divided into equal divisions of 15° about the noon line of the equatorial circle B_2G. The full construction for the hour lines of the dial is now readily completed by joining the appropriate divisions on the equinoctial line to the extremity A of the style at the dial plate.

A *totally* graphical construction for two dials, one declining East, the other West, is shown in FIGS. 38 and 39. Proceed as follows. Draw a horizontal line XX_1 and at any point X_2 erect a vertical X_2T. Now mark out the declination (60°E, Fig. 38, 28°W, FIG. 39) to the right of X_2T when the delination is East; to the left, when the declination is West so that we discover point X_3 on XX_1. Through X_3 draw a vertical $12 \cdot 12_1$ which is the noon line on the dial face. With the compasses at point X_3 and radius X_3T strike an arc to cut XX_1 in B_4. From B_4 strike off a line making an angle equal to the latitude (ϕ) with XX_1 to

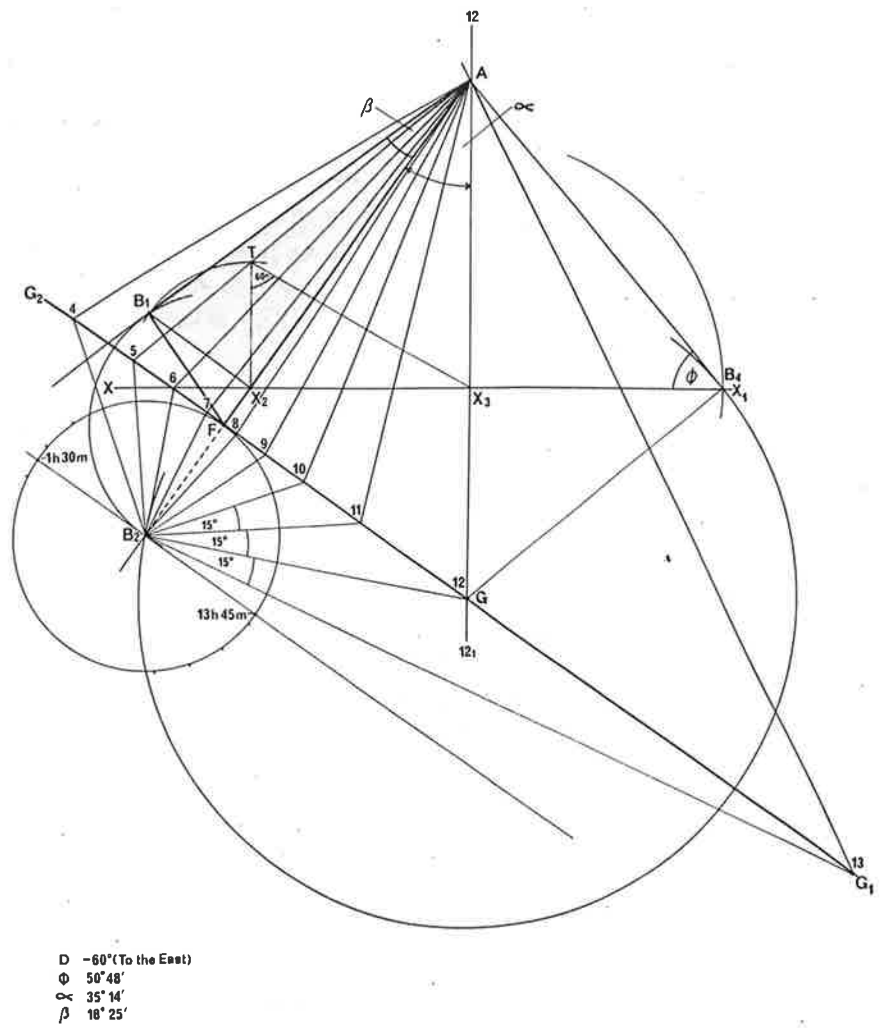


FIG. 38.

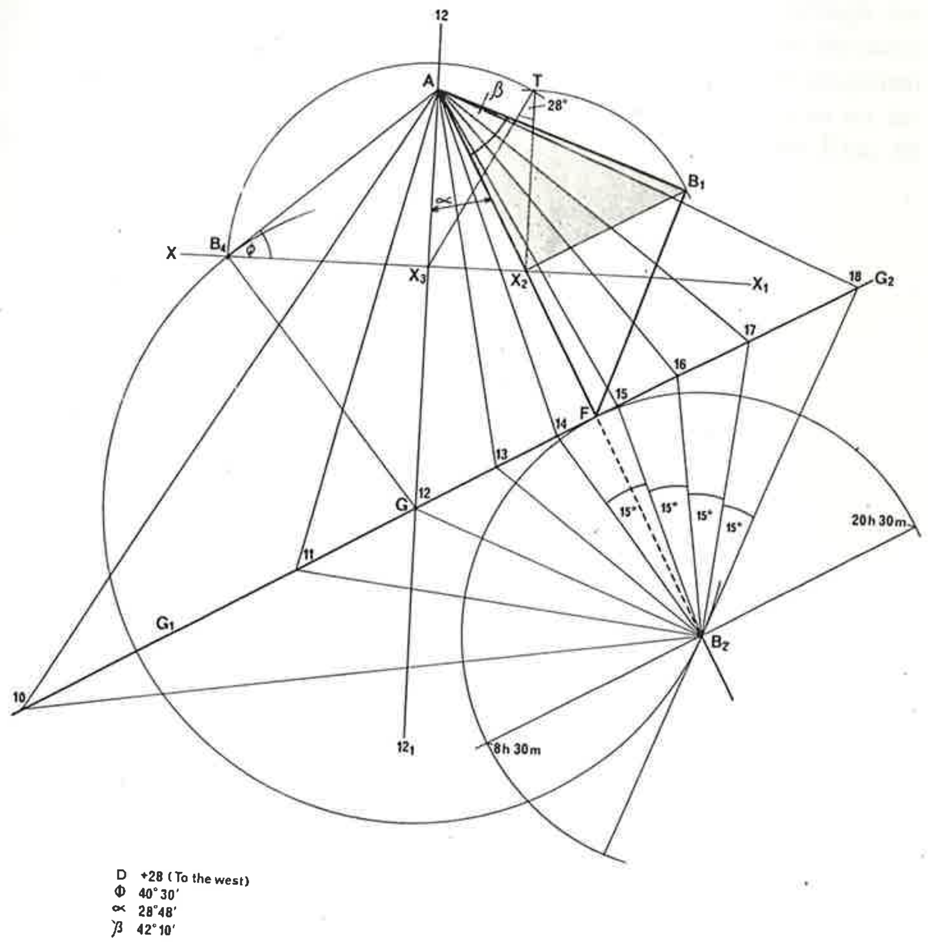


FIG. 39.

cut the $12 \cdot 12_1$ line at A. From B_4 draw a perpendicular to AB_4 to cut the $12 \cdot 12_1$ line in G. A line from A to X_2 extended gives the substyle line and a line normal to the substyle line through G gives the equinoctial line G_1G_2 and the intersection F on the substyle line. The centre of the equatorial dial B_2 is found on the substyle line by taking the radius GB_4 from G and striking an arc to cut AX_2 extended in B_2 ($B_4G = B_2G$). Again from the base line of the style a vertical is drawn from X_2 to the left (or the right, depending on the declination) having the same length as X_2T to obtain the extremity of the theoretical style B_1. If now A is joined to B_1 we have the theoretical style. To check the accuracy of the graphical layout measure FB_1 and FB_2: they must be identical. If not, an inaccuracy has been built up and the construction is vitiated. To draw the desired hour lines of the dials draw a circle, the equatorial circle, at centre B_2 with radius B_2F. Divide the circle into 24 equal divisions *about the line* B_2G and extend them to cut the equinoctial line G_1G_2. From these intersections draw lines to the style termination A on the dial plate. These lines are the desired hour lines. From a study of FIG. 40 it readily can be seen that the hour lines for any dial remain

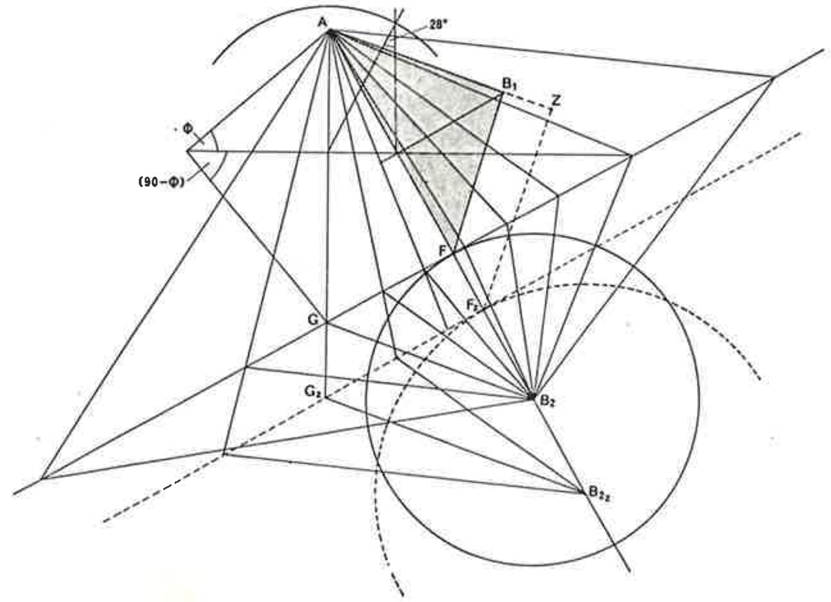


FIG. 40.

... clock time by making two corrections: ... west of the time meridian, the other for the equation of ... for the date of the observation.

unchanged as the theoretical style is increased in length from AB_1 to AZ_1.

From a study of the foregoing it will be clear that the technique applies with equal validity to the layout of a dial on a vertical face declining from true North. One illustration must suffice. Consider the case of the style shown in FIG. 41 declining East of south in a northern latitude (compare with FIG. 35.) A little consideration will show that the style on passing through the vertical face facing south-east comes out at the back at the same angle but on a face declining to the north-west. The graphical construction used for the southern face now appears as an inverted mirror image of itself for the northern face, see FIG. 42 and compare with FIG. 36.

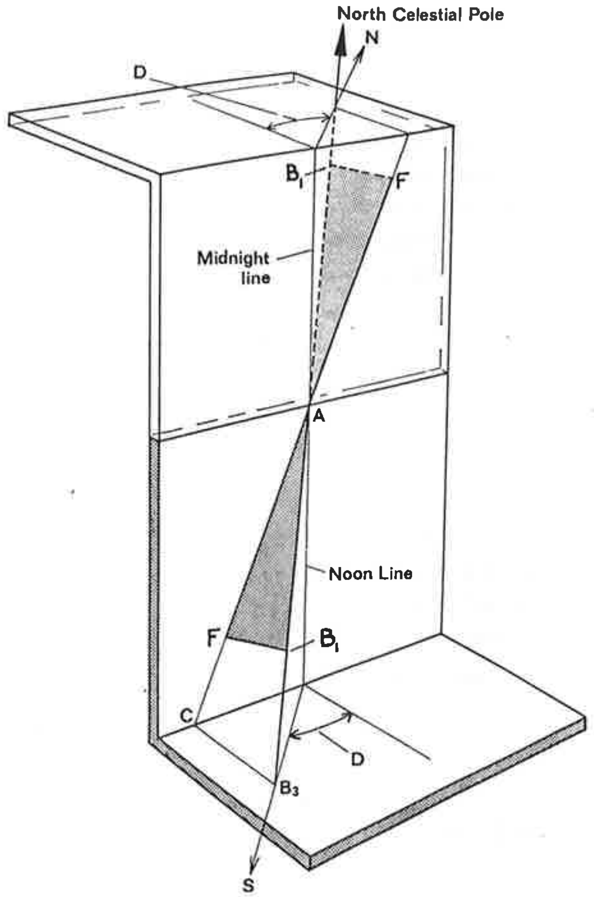


FIG. 41.

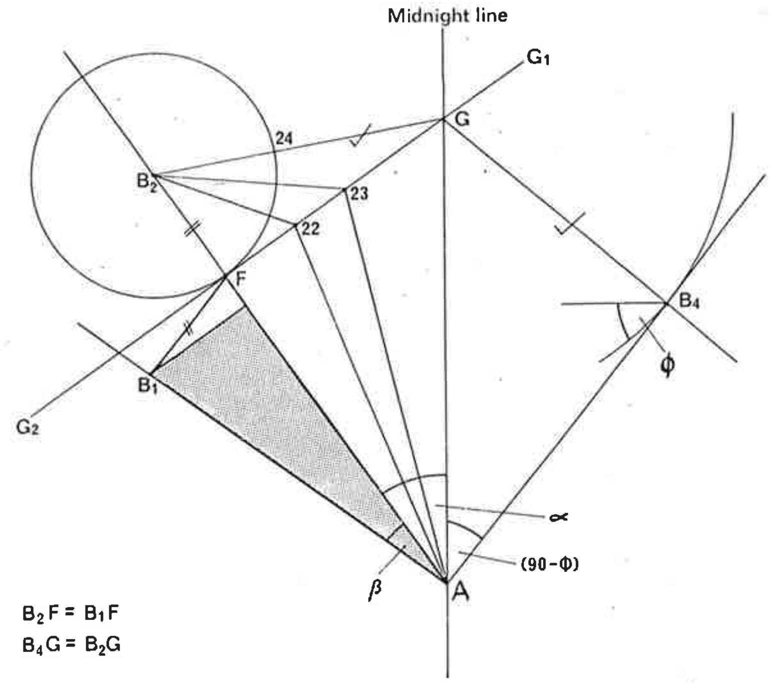


FIG. 42.

MATHEMATICAL CONSTRUCTION

The mathematics of these dials is now given by means of two fully worked examples. (See also the Appendix for a note on the accuracy of the computations.)

Let γ be the angle of the hour line with the substyle in the plane of the dial

- ϕ be the latitude of the sundial's station
- h be the Sun's hour angle
- H be the hour angle of the plane of the style (negative to the East, positive to the West)
- a be the angle between the substyle and the noon line in the plane of the dial
- β be the angle of the style with the substyle (in a plane normal to the plane of the dial)
- D be the declination or deviation of the normal to the plane of the dial (negative to the East, positive to the West)

Then
$$\tan \alpha = \sin D \cot \phi$$
$$\sin \beta = \cos D \cos \phi$$
$$\cot H = \cot D \sin \phi$$
$$\sin \alpha = \cos \phi \sin H$$
$$\tan \gamma = \cos D \cos \phi \tan (H-h), \text{ or}$$
$$\log \tan \gamma = \log \cos D + \log \cos \phi + \log \tan (H-h)$$

Example I

Declining 60°E in Latitude 50° 48′N (facing SE)

$\phi = 50° 48'$
$D = -60°$ (i.e. to the East)
$\alpha = 35° 14'$
$\beta = 18° 25'$
$H = -65° 54'$ (i.e. to the East)

$\tan \alpha = \sin D \cot \phi$
$= \sin 60° \cot 50° 48'$
$= 0{\cdot}86603 \times 0{\cdot}81558$
$= 0{\cdot}70632$
$\alpha = 35° 14'$

$\sin \beta = \cos D \cos \phi$
$= \cos 60° \cos 50° 48'$
$= 0{\cdot}5 \times 0{\cdot}63203$
$= 0{\cdot}31601$
$\beta = 18° 25'$

$\cot H = \cot D \sin \phi$
$= \cot 60 \sin 50° 48'$
$= 0{\cdot}57735 \times 0{\cdot}77494$
$= 0{\cdot}44741$
$H = 65° 54'$

The dial limits are $(H-h) = +90°$ or $-90°$
thus $h = -155° 54'$ or $+24° 06'$
i.e. -10h 24m or $+1$h 36m
i.e. 1h 36m and 13h 36m
apparent local time

Check calculation $\sin \alpha = \cos \phi \sin H$
$0{\cdot}5769 \simeq 0{\cdot}6320 \times 0{\cdot}91283$

Now $\log \tan \gamma = \log \cos D + \log \cos \phi + \log \tan (H-h)$
Note: $\log \cos D + \log \cos \phi = \log \cos 60° + \log \cos 50° 48'$
$= 9{\cdot}69897 + 9{\cdot}80074$
$= 9{\cdot}49971$

P

Hour h. m.	h	$H-h$	log tan $H-h$	log tan γ	γ
13.00	+15°	−80° 54′	10·79541	10·29512	−63° 07′
12.45	+11° 15′	−77° 09′	10·64185	10·14156	−54° 11′
12.30	+ 7° 30′	−73° 24′	10·52562	10·02533	−46° 40′
12.15	+ 3° 45′	−69° 39′	10·43074	9·93045	−40° 26′
Noon	0	−65° 54′	10·34938	9·84909	−35° 14′
11.45	− 3° 45′	−62° 09′	10·27707	9·77678	−30° 53′
11.30	− 7° 30′	−58° 24′	10·21098	9·71069	−27° 11′
11.15	−11° 15′	−54° 39′	10·14914	9·64885	−24° 01′
11.00	−15° 00′	−50° 54′	10·09008	9·58979	−21° 15′
10.45	−18° 45′	−47° 09′	10·03262	9·53233	−18° 49′
10.30	−22° 30′	−43° 24′	9·97573	9·47544	−16° 38′
10.15	−26° 15′	−39° 39′	9·91842	9·41813	−14° 41′
10.00	−30° 00′	−35° 54′	9.85967	9·35938	−12° 53′
9.45	−33° 45′	−32° 09′	9·79832	9·29803	−11° 14′
9.30	−37° 30′	−28° 24′	9·73295	9·23266	− 9° 42′
9.15	−41° 15′	−24° 39′	9·66171	9·16142	− 8° 15′
9.00	−45° 00′	−20° 54′	9·58191	9·08162	− 6° 53′
8.45	−48° 45′	−17° 09′	9·48939	8·98910	− 5° 34′
8.30	−52° 30′	−13° 24′	9·37700	8·87671	− 4° 18′
8.15	−56° 15′	− 9° 39′	9·23054	8·73025	− 3° 05′
8.00	−60° 00′	− 5° 54′	9·01427	8·51398	− 1° 52′
7.45	−63° 45′	− 2° 09′	8·57452	8·07423	− 0° 41′
7.30	−67° 30′	+ 1° 36′	8·44611	7·94582	+ 0° 30′
7.15	−71° 15′	+ 5° 21′	8·97150	8·47121	+ 1° 42′
7.00	−75° 00′	+ 9° 06′	9·20459	8·70430	+ 2° 54′
6.45	−78° 45′	+12° 51′	9.35815	8·85786	+ 4° 07′
6.30	−82° 30′	+16° 36′	9·47438	8·97409	+ 5° 23′
6.15	−86° 15′	+20° 21′	9·56926	9·06987	+ 6° 41′
6.00	−90° 00′	+24° 06′	9·65062	9·15033	+ 8° 03′
5.45	−93′ 45′	+27° 51′	9·72293	9·22264	+ 9° 29′
5.30	−97° 30′	+31° 36′	9·78902	9·28873	+11° 00′
5.15	−101° 15′	+35° 21′	9·85086	9·35057	+12° 38′
5.00	−105° 00′	+39° 06′	9·90992	9·40963	+14° 24′
4.45	−108° 45′	+42° 51′	9·96738	9·46709	+16° 21′
4.30	−112° 30′	+46° 36′	10·02427	9·52398	+18° 29′
4.15	−116° 15′	+50° 21′	10·08158	9·58129	+20° 52′
4.00	−120° 00′	+54° 06′	10·14033	9·64004	+23° 35′

NOTE: For ease of tabulation and printing negative characteristics are rejected. The number 10 is added to all the logarithms in the table. Hence 9 denotes $\bar{1}$, 8 denotes $\bar{2}$, 10 denotes 0. Logarithms thus increased are called 'tabular logarithms'.
tabular logarithm = true logarithm + 10
For a note on the accuracy of these tabulated values see the appendix.
NOTE: further these figures apply to a dial facing NW on the wall opposite to the wall facing SE. See figure 31.

Example II

Declining 28°W in Latitude 40° 30′N (facing SW)

$\phi = 40° 30'$
$D = +28°$ (i.e. to the West)
$\alpha = 28° 48'$
$\beta = 42° 10'$
$H = +39° 18'$ (i.e. to the West)

Holwell "P"
Waugh p. 50
"P"

$$\begin{aligned} \tan \alpha &= \sin D \cot \phi \\ &= \sin 28^\circ \cot 40^\circ 30' \\ &= 0{\cdot}46947 \times 1{\cdot}17085 \\ &= 0{\cdot}54968 \\ \alpha &= 28^\circ 48' \end{aligned}$$

$$\begin{aligned} \sin \beta &= \cos D \cos \phi \\ &= \cos 28^\circ \cos 40^\circ 30' \\ &= 0{\cdot}88295 \times 0{\cdot}76041 \\ &= 0{\cdot}67140 \\ \beta &= 42^\circ 10' \end{aligned}$$

$$\begin{aligned} \cot H &= \cot D \sin \phi \\ &= \cot 28^\circ \sin 40^\circ 30' \\ &= 1{\cdot}88073 \times 0{\cdot}64945 \\ &= 1{\cdot}22144 \\ H &= 39^\circ 18' \end{aligned}$$

Check calculation $\sin \alpha = \cos \phi \sin H$

$0{\cdot}48175 \simeq 0{\cdot}7604 \times 0{\cdot}63338$

The dial limits are

$(H-h) = +90^\circ$ or -90°

$h = -50^\circ 42'$ or $+129^\circ 18'$

i.e. -3h 23m or $+8$h 37m

i.e. 8h 37m and 20h 37m apparent local time (see FIG. 39)

Now $\log \tan \gamma = \log \cos D + \log \cos \phi + \log \tan (H-h)$

Note: $\log \cos D + \log \cos \phi = \log \cos 28^\circ + \log \cos 40^\circ 30'$

$= 9{\cdot}94593 + 9{\cdot}88105$

$= 9{\cdot}82698$ which is constant

Hour h. m.	h°	$H-h$	log tan $H-h$	log tan γ	γ
20.00	+120° 00′	−80° 42′	10·78580	10·61278	−76° 18′
19.45	+116° 15′	−76° 57′	10·63491	10·46189	−70° 57′
19.30	+112° 30′	−73° 12′	10·52011	10·34709	−65° 47′
19.15	+108° 45′	−69° 27′	10·42611	10·25309	−60° 49′
19.00	+105° 00′	−65° 42′	10·34533	10·17231	−56° 05′
18.45	+101° 15′	−61° 57′	10·27341	10·10039	−51° 34′
18.30	+97° 30′	−58° 12′	10·20759	10·03457	−47° 17′
18.15	+93° 45′	−54° 27′	10·14593	9·97291	−43° 13′
18.00	+90° 00′	−50° 42′	10·08699	9·91397	−39° 22′
17.45	+86° 15′	−46° 57′	10·02958	9·85656	−35° 42′
17.30	+82° 30′	−43° 12′	9·97269	9·79967	−32° 14′
17.15	+78° 45′	−39° 27′	9·91533	9·74231	−28° 55′
17.00	+75° 00′	−35° 42′	9·85647	9·68345	−25° 45′
16.45	+71° 15′	−31° 57′	9·79495	9·62193	−22° 43′
16.30	+67° 30′	−28° 12′	9·72932	9·55630	−19° 48′
16.15	+63° 45′	−24° 27′	9·65770	9·48468	−16° 59′
16.00	+60° 00′	−20° 42′	9·57734	9·40432	−14° 14′
15.45	+56° 15′	−16° 57′	9·48398	9·31096	−11° 33′
15.30	+52° 30′	−13° 12′	9·37023	9·19721	− 8° 57′
15.15	+48° 45′	− 9° 27′	9·22127	9·04825	− 6° 23′
15.00	+45° 00′	− 5° 42′	8·99919	8·82617	− 3° 50′
14.45	+41° 15′	− 1° 57′	8·53208	8·35906	− 1° 19′
14.30	+37° 30′	+ 1° 48′	8·49729	8·32427	+ 1° 12′
14.15	+33° 45′	+ 5° 33′	8·98753	8·81451	+ 3° 44′
14.00	+30° 00′	+ 9° 18′	9·21420	9·04118	+ 6° 17′
13.45	+26° 15′	+13° 03′	9·36509	9·19207	+ 8° 51′
13.30	+22° 30′	+16° 48′	9·47989	9·30687	+11° 27′
13.15	+18° 45′	+20° 33′	9·57389	9·40087	+14° 07′
13.00	+15° 00′	+24° 18′	9·65467	9·48165	+16° 52′
12.45	+11° 15′	+28° 03′	9·72659	9·55357	+19° 41′
12.30	+ 7° 30′	+31° 48′	9·79241	9·61939	+22° 36′
12.15	+ 3° 45′	+35° 33′	9·85407	9·68105	+25° 38′
Noon	0	+39° 18′	9·91301	9·73999	+28° 47′
11.45	− 3° 45′	+43° 03′	9·97042	9·79740	+32° 06′
11.30	− 7° 30′	+46° 48′	10·02731	9·85429	+35° 34′
11.15	−11° 15′	+50° 33′	10·08467	9·91165	+39° 13′
11.00	−15° 00′	+54° 18′	10·14353	9·97051	+43° 03′
10.45	−18° 45′	+58° 03′	10·20505	10·03203	+47° 07′
10.30	−22° 30′	+61° 48′	10·27068	10·09766	+51° 23′
10.15	−26° 15′	+65° 33′	10·34230	10·16928	+55° 54′
10.00	−30° 00′	+69° 18′	10·42266	10·24964	+60° 38′

For a note on these tabulated values see the Appendix.

°	h.	m.	°	h.	m.	°	h.	m.	′	″	m.	s.	′	″	m.	s.	′	″	m.	s.	′	″	m.	s.
44	2	56	104	6	56	164	10	56	11	00	0	44	26	00	1	44	41	00	2	44	56	00	3	44
45	3	0	105	7	0	165	11	0		15	0	45		15	1	45		15	2	45		15	3	45
46	3	4	106	7	4	166	11	4		30	0	46		30	1	46		30	2	46		30	3	46
47	3	8	107	7	8	167	11	8		45	0	47		45	1	47		45	2	47		45	3	47
48	3	12	108	7	12	168	11	12	12	00	0	48	27	00	1	48	42	00	2	48	57	00	3	48
49	3	16	109	7	16	169	11	16		15	0	49		15	1	49		15	2	49		15	3	49
50	3	20	110	7	20	170	11	20		30	0	50		30	1	50		30	2	50		30	3	50
51	3	24	111	7	24	171	11	24		45	0	51		45	1	51		45	2	51		45	3	51
52	3	28	112	7	28	172	11	28	13	00	0	52	28	00	1	52	43	00	2	52	58	00	3	52
53	3	32	113	7	32	173	11	32		15	0	53		15	1	53		15	2	53		15	3	53
54	3	36	114	7	36	174	11	36		30	0	54		30	1	54		30	2	54		30	3	54
55	3	40	115	7	40	175	11	40		45	0	55		45	1	55		45	2	55		45	3	55
56	3	44	116	7	44	176	11	44	14	00	5	56	29	00	1	56	44	00	2	56	59	00	3	56
57	3	48	117	7	48	177	11	48		15	0	57		15	1	57		15	2	57		15	3	57
58	3	52	118	7	52	178	11	52		30	0	58		30	1	58		30	2	58		30	3	58
59	3	56	119	7	56	179	11	56		45	0	59		45	1	59		45	2	59		45	3	59
60	4	0	120	8	0	180	12	0	15	00	1	0	30	00	2	0	45	00	3	0	60	00	4	0

PROOFS OF FORMULAE

The formulae for the various forms of sundial are most readily derived by spherical trigonometry.

The celestial sphere (FIG. A.1) is an imaginary sphere of any size surrounding the observer at its centre O. Any plane through O will cut the surface of the sphere in a great circle, and the figure formed by the intersection of three great circles is a spherical triangle, the simplest form of which will have two sides at right angles. In a spherical triangle ABC, right-angled at C, we have

$$\sin a = \sin c \sin A$$
$$\cos a \sin b = \sin c \cos A$$
$$\cos a \cos b = \cos c$$

From the first two equations

$$\tan a = \tan A \sin b$$

which is the form most frequently used in this work. In these equations we may exchange A and a for B and b respectively, giving two more equations and leading to a complete solution of the triangle.

In FIG. A.1, P is the pole, Z the observer's zenith, NZS his meridian and NWS the plane of his horizon, OP is the extended style of the sundial, NP being equal to the latitude ϕ of the sundials' station. The plane of the shadow of the style is PVH,

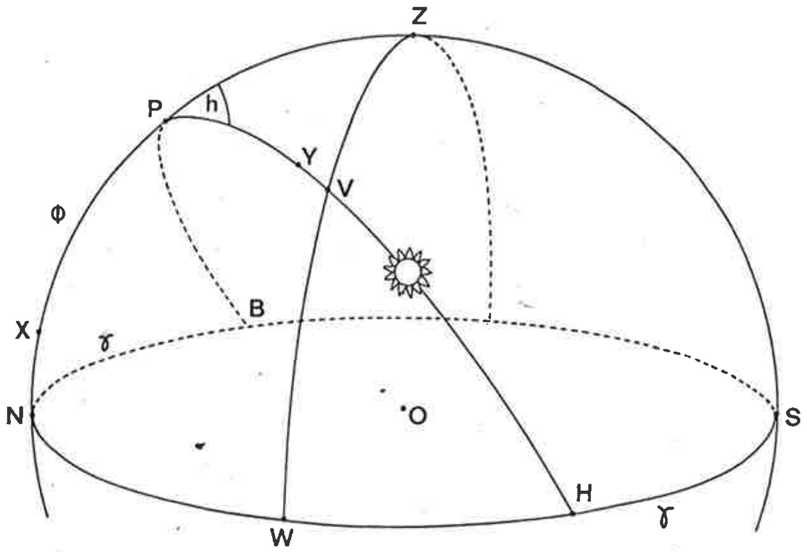


FIG. A1.

and the Sun must lie in this plane, which cuts the horizon at H and the vertical (east-west) plane at V. The hour-angle of the Sun is the angle $ZPV = h$.

In the *horizontal dial* γ is the angle between the substyle ON and the edge of the shadow OB, so that $\gamma = NB = SH$. Hence in the triangle PNH, right-angled at N (FIG. A.2), we have

$$\tan (180° - \gamma) = \tan (180° - h) \sin \phi$$
from which $\tan \gamma = \tan h \sin \phi$.

In the *horizontal reclining dial*, which is tilted at an angle R to the south, the plane of the dial is now XW, where $NX = R$. As explained in the text, this is exactly equivalent to a horizontal dial in latitude $\phi - R$, so that

$$\tan \gamma = \tan h \sin (\phi - R).$$

In the *vertical* (*south-facing*) *dial* the plane of the dial is ZVW, and γ is now the angle between the substyle OZ and the edge of the shadow OV, so that $\gamma = ZV$. In the triangle PZV, right-angled at Z (FIG. A.3)

$$\tan \gamma = \tan h \sin (90° - \phi)$$
$$\tan \gamma = \tan h \cos \phi$$

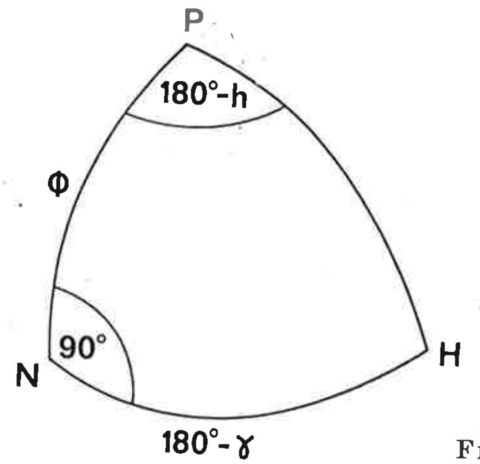


FIG. A2.

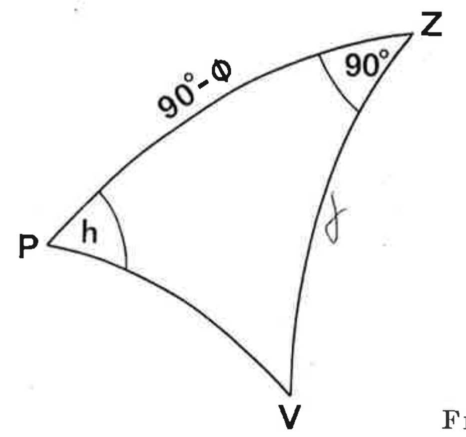
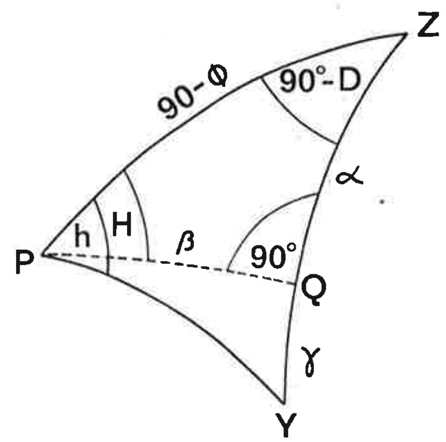


FIG. A3.



IG. A4.

SUNDIALS

In the *vertical declining dial* the vertical plane of the dial is no longer ZV but is now ZY, which declines to the west at an angle D, so that the angle PZY is $90° - D$ (see FIG. A.4). The style in this case is perpendicular to the face of the dial and is represented by PQ, and this plane is at an hour angle H. The angle α between the substyle and the noon-line is ZQ, and the angle β between the style and the substyle is PQ. Then in the triangle PZQ, right-angled at Q, we may use all five of the standard formulae:

? gnomon

$$
\begin{array}{lll}
\sin\beta & = \sin(90° - \phi)\sin(90° - D) & = \cos\phi\cos D \quad \ldots (1) \\
\cos\beta\sin\alpha & = \sin(90° - \phi)\cos(90° - D) & = \cos\phi\sin D \quad \ldots (2) \\
\cos\beta\cos\alpha & = \cos(90° - \phi) & = \sin\phi \quad \ldots (3) \\
\sin\alpha & = \sin(90° - \phi)\sin H & = \cos\phi\sin H \quad \ldots (4) \\
\sin\beta\cos\alpha & = \sin(90° - \phi)\cos H & = \cos\phi\cos H \quad \ldots (5)
\end{array}
$$

Equation (1) gives the value of β:

$$\sin\beta = \cos\phi\cos D$$

Dividing (2) by (3) we have for the value of α:

$$\tan\alpha = \cot\phi\sin D$$

Dividing (4) by (5) gives an equation for H:

$$\tan H = \tan\alpha/\sin\beta = \tan D/\sin\phi$$

In this dial, γ is the angle between the substyle OQ and the edge of the shadow OY, so that $\gamma = QY$. In the triangle PQY, right-angled at Q,

$$
\begin{aligned}
\tan\gamma &= \tan(h - H)\sin\beta \\
&= \tan(h - H)\cos\phi\cos D.
\end{aligned}
$$

In all of these formulae, it is the usual convention to take the angles h, H and D as positive if measured to the west, negative to the east, but no regard has been paid to the sign of γ. It must be remembered that γ is always marked off from the substyle on the side opposite to the Sun, i.e., γ has the opposite sign to h or $h - H$.

J.G.P.